ON THE DESIGN OF MATCHED EQUALIZERS
OF PRESCRIBED GAIN VERSUS FREQUENCY PROFILES

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Abstract

Design equations are derived for matched microstrip equalizers which provide a specified gain at three passband frequencies and present a good match to both source and load. Design examples compare measured results to theoretical predictions, and the application of matched equalizers to microwave amplifier design is discussed.

Introduction

Lossless equalizers can be designed to any desired gain versus frequency slope.[1-2] These reactive equalizers provide compensation by reflecting unwanted available power and are therefore not satisfactory in applications where a good match is to be presented to both source and load. Here the relevant equations are obtained for the design of a specific type of matched equalizer which is conveniently realized in microstrip form. These matched equalizers are designed to have an exact match at the passband edges (ω_U and ω_L) and an approximate match within the passband ($\omega_L < \omega < \omega_U$). The loss of these equalizers is 0 dB at the upper cut-off (ω_U), can have any specified loss at the lower cut-off (ω_L) and a loss which can be approximately specified at any frequency within the passband.

Equations for the gain and matching of the basic equalizer structure of Figure 1 are first developed. These equations are then made more specific as they are applied to the resistor-transmission line circuit of Figure 2 and finally to a circuit convenient for microstrip realization (Figure 3).

Design steps are outlined and illustrated through specific design examples and measured results are compared to theoretical predictions. Finally, the application of matched equalizers to microwave amplifier design is discussed and illustrated.

Development of Design Equations

The equations relevant to the equalizer design are here first developed in terms of the basic equalizer structure of Figure 1, next applied to Figure 2 and finally to the circuit of Figure 3. The transducer gain for Figure 1 can be written as follows:

$$G_T = \frac{P_L}{P_{AVS}} = \frac{4\,|y_1|^2}{|(y_2 + y_1 + 1)^2 - y_1^2|^2} \tag{1}$$

(General Equation for Equalizer Transducer Gain)

The requirement of a good input and output match ($y_{in}=y_{out}=1$) requires a favorable relationship between y_1 and y_2. The relationship between y_1 and y_2 to assure matching of the equalizer is:

$$y_1 = \frac{1 - y_2^2}{2y_2} \tag{2}$$

(Condition for Matched Equalizer)

We will apply equation (2) to assure a perfect match at the lower end of the passband (ω_L). Further, since the resonators of Figures 2 and 3 are all tuned to ω_U), a perfect match at ω_U is also assured. This matched condition at the band edges (ω_L and ω_U) implies that the equalizer will be reasonably well matched and that equation (2) will approximately hold throughout the passband ($\omega_L < \omega < \omega_U$). This assumption of good matching throughout the passband allows the simplification of equation (1) under the assumption that (2) holds. The resultant transducer gain is called the matched transducer gain (G_{TM}) and is obtained by substituting (2) into (1):

$$G_{TM} = \frac{|1 - y_2|^2}{|1 + y_2|^2} \tag{3}$$

(Transducer Gain of Matched Equalizer)

This is the matched transducer gain and will be used in the design procedure to obtain an exactly prescribed loss at ω_L and an approximately prescribed loss at ω_M ($\omega_L < \omega_M < \omega_U$).

Equations (2) and (3) will now be applied to the circuit of Figure 2 to obtain an equalizer to the following specifications:

a) an exact match at ω_L
b) a specified gain at ω_L: $G_T(\omega_L)$
c) an approximately specified gain at ω_M ($\omega_L<\omega_M<\omega_U$): $G_T(\omega_M)$.

The matching condition at ω_L results in:

$$G_1 + j(1/Z_{01})\tan(\omega_L\tau) = \frac{1}{2}\left[R_2 - \frac{R_2}{R_2^2 + X_L^2}\right] + \frac{j}{2}\left[X_L + \frac{X_L}{R_2^2 + X_L^2}\right] \tag{4}$$

where $X_L = Z_{02}\tan(\omega_L\tau)$ and $\tau = \frac{\pi}{2\omega_U}$

The gain specification at ω_L results in:

$$G_T(\omega_L) = \frac{(R_2-1)^2 + X_L^2}{(R_2+1)^2 + X_L^2} \tag{5}$$

The gain specification at ω_M results in the approximate relation:

$$G_T(\omega_M) \cong \frac{(R_2-1)^2 + X_M^2}{(R_2+1)^2 + X_M^2} \tag{6}$$

where $X_M = Z_{02}\tan(\omega_M\tau)$

Equations (5) and (6) provide a simultaneous solution for R_2 and Z_{02}. If these equations are combined to eliminate Z_{02}, a quadratic equation in R_2 results:

$$AR_2^2 + BR_2 + C = 0$$

$$A = C = (G_T(\omega_L) - 1 - K\, G_T(\omega_M) + K)$$

$$B = 2(G_T(\omega_L) + 1 - K\, G_T(\omega_M) - K) \tag{7}$$

$$K = \frac{(1 - G_T(\omega_L))\, \tan^2(\omega_L \tau)}{(1 - G_T(\omega_M))\, \tan^2(\omega_M \tau)}$$

Equations (4) through (7) are sufficient for obtaining R_2, Z_{02}, G_1 and Z_{01} of Figure 2. The series L - C realization of Z_{01} can be obtained by requiring that the L - C resonator have the same admittance as Z_{01} at ω_L and ω_U. The equations resulting from this requirement are as follows:

$$L = \frac{Z_{01}}{\omega_U \tan(\omega_L \tau)[\omega_U/\omega_L - \omega_L/\omega_U]} \tag{8}$$

$$C = \frac{1}{\omega_U{}^2 L}$$

Equations (4) through (8) are sufficient for the design of the equalizer of Figure 3 to a prescribed $G_T(\omega_L)$ and $G_T(\omega_M)$.

Outline of Equalizer Design Procedure

The following specific design steps outline the procedure for obtaining L, C, Z_{02} and R_2 of Figure 3 from a desired $G_T(\omega_L)$ and $G_T(\omega_M)$.

Step 1: Obtain value(s) of R_2 from equation (7).

Step 2: Apply equation (5) to obtain the value(s) of Z_{02}.

Step 3: Use equation (4) to determine value(s) of G_1 and Z_{01}.

Step 4: Scale all values of impedance from 1 ohm source and load to the desired impedance level.

Step 5: Obtain L and C for the tuned circuit realization of Z_{01} in Figure 3 by application of equation (8).

Step 6: If steps 1 through 5 have not produced a realizable solution, $G_T(\omega_M)$ and/or $G_T(\omega_L)$ should be respecified and steps 1 through 5 repeated.

Design Examples

The design examples are presented here following the numbered steps outlined in the previous section.

Example 1: An equalizer is to be designed for operation over 5 to 13 GHz in a 50Ω system. The desired losses are 6 dB at 5 GHz, 3 dB at 8.06 GHz and 0 dB at 13 GHz.

Step 1: Equation (7) gives $R_2 = .473$ and $R_2 = 2.11$.

Step 2: Equation (5) produces $Z_{02} = .866$ (for $R_2 = .473$) and $Z_{02} = 1.83$ (for $R_2 = 2.11$).

Step 3: Equation (4) gives $G_1 = -.170$ and $Z_{01} = .849$ (for $R_2 = .473$, $Z_{02} = .866$) and this solution is discarded.
For $R_2 = 2.11$ and $Z_{02} = 1.83$ Equation (5) produces $G_1 = .882$ and $Z_{01} = .938$.

Step 4: Impedance values are scaled to the 50 ohm level:
$R_2 = 106\Omega$, $Z_{02} = 91.5\Omega$
$G_1 = .0176$ mho and $Z_{01} = 46.9\Omega$.

Step 5: Equation (8) gives L = .375 nH and C = .399 pF.

This design was realized as shown in Figure 4 and tested on a network analyzer. Figure 5 compares measured versus predicted results.

Example 2: A 50 ohm equalizer is to be designed to operate between 5 and 13 GHz with 3 dB loss at 5 GHz, 1.5 dB loss at 9 GHz, and 0 dB loss at 13 GHz.

With G_T (9 GHz) = —1.5 dB, steps 1 through 4 produce a solution with $R_2 = 248$, $Z_{02} = 149$, $G_1 = .0479$ and $Z_{01} = 32.35$. Since the 149 ohm characteristic impedance is difficult to achieve in microstrip, the problem is respecified with G_T (9 GHz) = —2.0 dB. Steps 1 through 4 produce a solution with R = 270, $Z_{02} = 110$, $G_1 = .0524$ and $Z_{01} = 44.1$. Step 5 produces a series resonant circuit approximating Z_{01} with L = .353 nH and C = .425 pF.

The response of this design and comparison to theoretical data is shown in Figure 6.

Application of Matched Equalizers in Microwave Amplifier Design

The gain roll-off with frequency of microwave transistors can be compensated by lossless coupling networks which exhibit a gain roll-up with frequency[2]. These lossless networks achieve compensation by reflecting unwanted available power and do not provide a good match to the components with which they are cascaded. The matched (lossy) compensators described here, however, can compensate the gain roll-off of microwave transistors and still present a good input and output match. Figure 7 illustrates the methods of compensation. In Figure 7a the 6 dB/octave gain roll-up required to compensate the GaAs FET is accomplished by a lossless (reflective) compensator and results in a high input VSWR at low frequencies (Figure 7c). In Figure 7b a flat (no compensation), input matching network matches the input impedance of the FET to 50Ω and the matched 50Ω compensator provides the 6 dB/octave compensation while maintaining a good 50Ω input and output impedance. The good input VSWR obtained by the lossy (matched) compensation amplifier makes the resultant amplifier readily cascadeable.

Conclusions

The design equations here presented provide a direct method for obtaining matched compensators of desired gain versus frequency profiles which are compactly realizable in microstrip form. These matched compensators present a good match to both source and load and therefore are easily cascadeable system building blocks. They provide a convenient means of compensating the gain roll-off of microwave amplifiers and provide well matched, easily cascadeable amplifier gain stages.

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References

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2. D. J. Mellor and J. G. Linvill, "Synthesis of Interstage Networks of Prescribed Gain Versus Frequency Slopes", IEEE Trans. Microwave Theory Tech., vol. MTT-23, pp. 1013-1020, Dec. 1975.

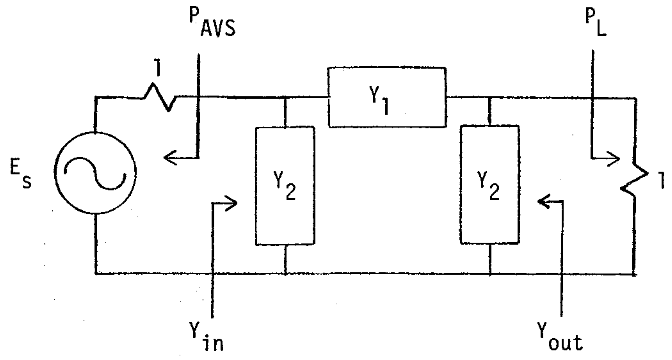


Figure 1. Basic Equalizer Structure

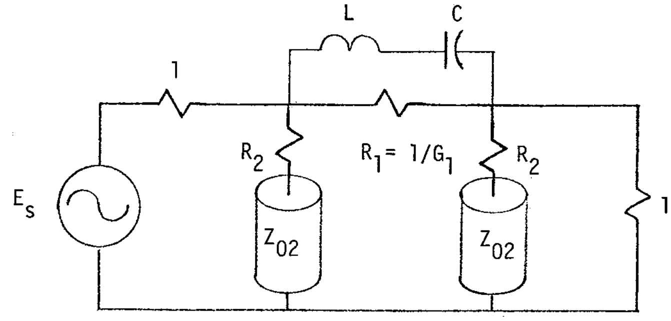


Figure 3. Equalizer with series resonant realization of series line Z_{01}. All lines are $\lambda/4$ at ω_U and L and C are resonant at ω_U.

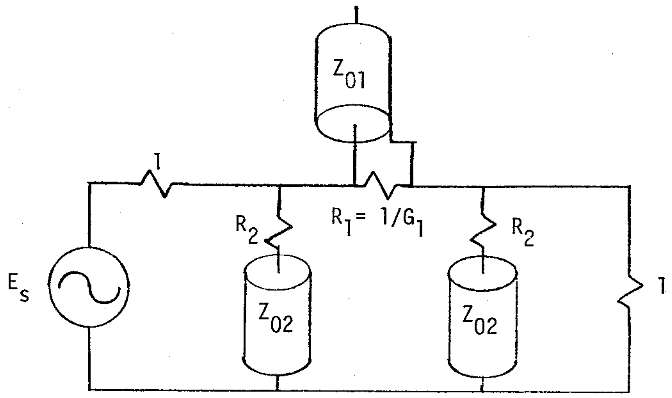


Figure 2. Resistor-Transmission Line Equalizer. All lines are $\lambda/4$ at ω_U.

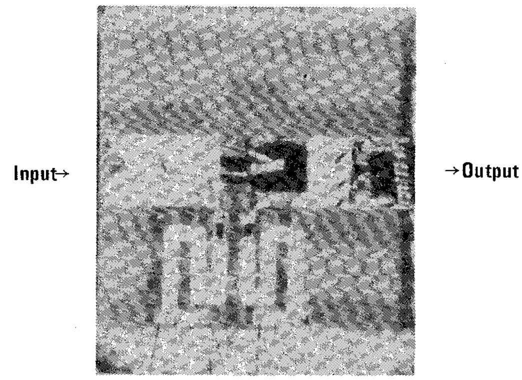


Figure 4. Construction Technique for Matched Microstrip Equalizers.

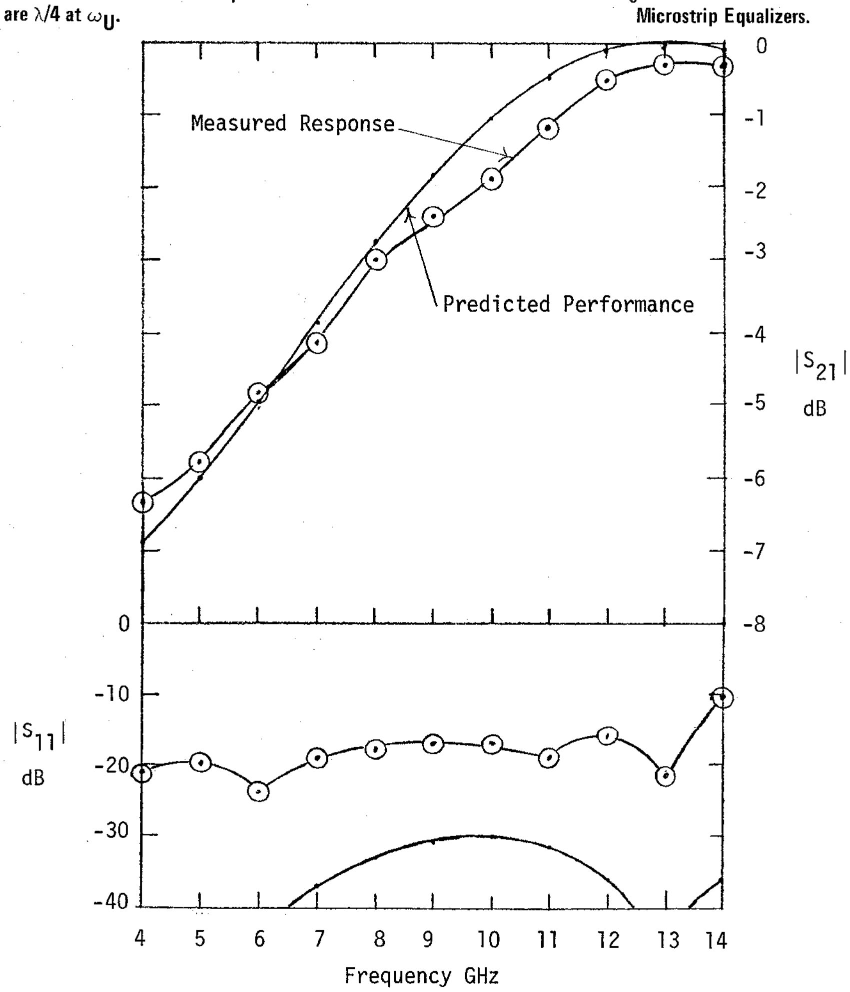


Figure 5. Matched Equalizer Providing 6 dB of Compensation from 5 to 13 GHz.

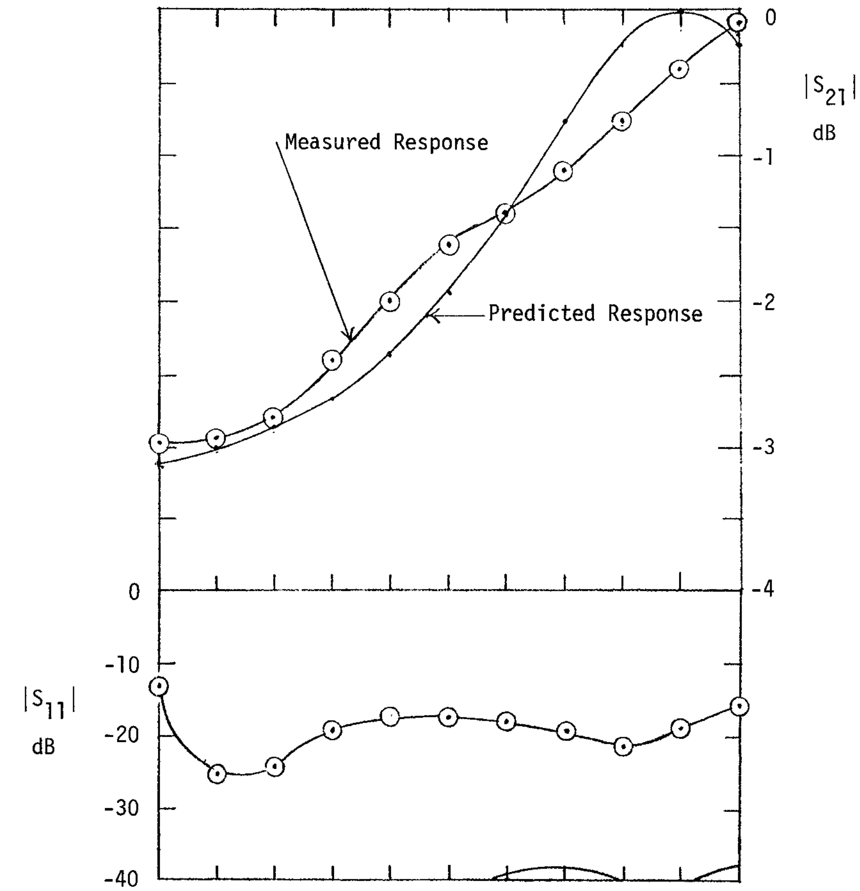
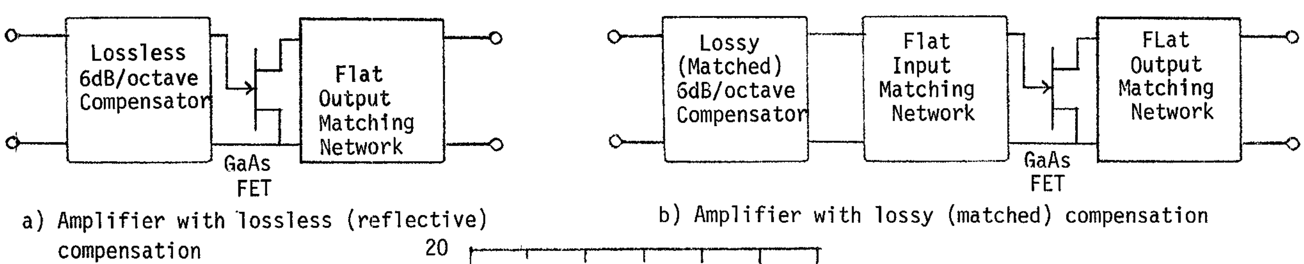


Figure 6. Matched Equalizer Providing 3 dB of Compensation from 5 to 13 GHz.



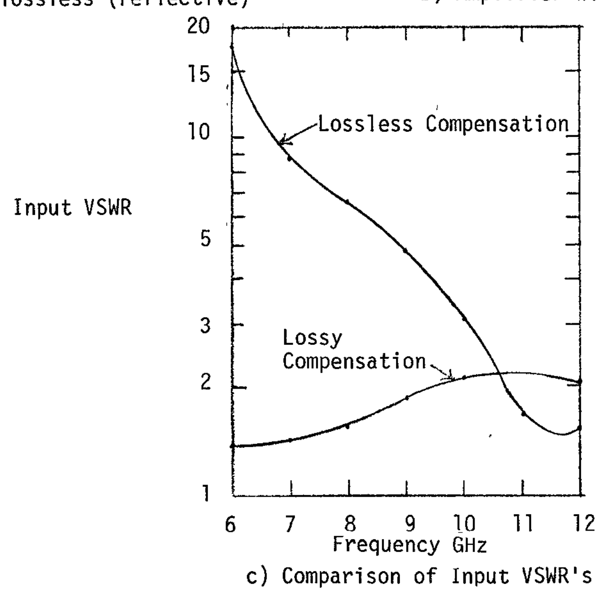


Figure 7. Comparison of Lossless and Lossy Compensation Methods for Microwave Amplifiers.